Fault Detection Over Wireless Sensor Networks using Distributed Kalman and Distributed Particle Filtering

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1 Introduction

The chapter considers the problem of distributed fault detection and isolation for continuous time dynamical systems. Such a fault diagnosis procedure involves the transmission of measurements to local processing units over a wireless sensor network and the fusion of local state estimates with the use of distributed filtering algorithms. Most of the existing fault diagnosis methods are centralized, which means that all sensing data are collected and processed at one unit. This not only causes excessive communication and computational burdens, but also creates a single point of failure. To overcome these problems, distributed detection and data fusion methods are being developed. Distributed fault diagnosis can be performed by: (a) running local fault diagnosers (hypotheses tests) at distributed processing units, obtaining local diagnosis results and fusing the local diagnoses so as to reach the final decision on the existence of a fault (Chamberland, 2003; Nguyen et al., 2008), (b) fusing the individual state estimates provided by distributed filters and using the aggregate state vector in a fault diagnosis algorithm to generate residuals (Okatan et al., 2001; Salahshoor et al., 2008). In this Chapter the problem of distributed fault diagnosis will be studied according to second approach.

In many applications it is possible to obtain the state vector of a dynamical system simultaneously from distributed sources. For example: (i) in the aerospace and maritime systems when transmitting measurements about the system’s condition to distributed processing units, (ii) in chemical processes when measuring the system’s state variables with multiple sensors and dispatching these measurements to local processing units. Based on these measurements, filtering algorithms running on different processing units produce estimates of the system’s state vector while to improve the estimation accuracy and the reliability of data processing, fusion of the distributed state estimates is performed by an aggregation filter. The overall state estimation procedure is known as distributed filtering (Rigatos, 2011).

The chapter first proposes the Extended Information Filter (EIF) and the Unscented Information Filter (UIF) as possible distributed filtering approaches which enable to obtain an estimation of the state vector of the monitored system, under the assumption of Gaussian noises (Nettleton et al., 2003; Lee, 2008; Vercauteren, 2005). Next, the Distributed Particle Filter (DPF) is proposed as a distributed filtering method which is well-suited for providing estimates of the monitored system’s state vector in the case of non-Gaussian measurements (Rigatos, 2009a). Efficient implementation of fusion of the local probability density functions associated with the local Particle Filters are analyzed (Musso et al., 2001). The state vector which is estimated with the use of the EIF, UIF or DPF can be used again by an FDI algorithm for residuals generation. After residuals have been...
produced a remaining problem in particle filtering-based FDI algorithms is the definition of an effective fault threshold based on a suitable reformulation of the Generalized Likelihood Ratio (Li and Kadirkamanathan, 2001; Rigatos, 2009b).

The Extended Information Filter is a generalization of the Information Filter in which the local filters do not exchange raw measurements but send to an aggregation filter their local information matrices (local inverse covariance matrices which can be also associated to the Fisher Information Matrices) and their associated local information state vectors (products of the local information matrices with the local state vectors) (Nettleton et al., 2003; Lee, 2008; Vercauteren, 2005). In the case of the Unscented Information Filter there is no linearization of the system’s observation equation. However the application of the Information Filter algorithm is possible through an implicit linearization which is performed by approximating the Jacobian matrix of the system’s output equation by the product of the inverse of the state vector’s covariance matrix (information matrix) with the cross-correlation covariance matrix between the system’s state vector and the system’s output (Lee, 2008; Vercauteren, 2005). Again, the local information matrices and the local information state vectors are transferred to an aggregation filter which produces the global estimation of the system’s state vector. The EIF and UIF estimated state vector can in turn be used by a FDI algorithm for residuals generation. After residuals have been obtained a remaining problem in the FDI procedure is the definition of an effective fault threshold based on the Likelihood Ratio or the Generalized Likelihood Ratio (Basseville and Nikiforov, 1993; Rigatos and Zhang, 2009).

Next, the Distributed Particle Filter (DPF) is proposed as a distributed filtering method which is well-suited for providing estimates of the monitored system’s state vector in the case of non-Gaussian measurements (Rigatos, 2009a). Difficulties in implementing distributed particle filtering stem from the fact that particles from one particle set (which correspond to a local particle filter) do not have the same support (do not cover the same area and points on the samples space) as particles from another particle set (which are associated with another particle filter) (Rigatos, 2011; Ong et al., 2008; Snoussi, 2006). This can be resolved by transforming the particles sets into Gaussian mixtures, and defining the global probability distribution on the common support set of the probability density functions associated with the local filters (Musso et al., 2001). The state vector which is estimated with the use of the DPF can be used again by the FDI algorithm for residuals generation. After residuals have been produced a remaining problem in particle filtering-based FDI algorithms is the definition of an effective fault threshold based on a suitable reformulation of the Generalized Likelihood Ratio (Li and Kadirkamanathan, 2001; Rigatos, 2009b).

This chapter is organized as follows: Section 2 analyzes the problem of Distributed Fault Diagnosis. Section 3 studies the Extended Information Filter (Distributed Extended Kalman Filter). In Section 4, the Unscented Information Filter (Distributed Unscented Kalman Filter) is analyzed and its use for fusing distributed state estimates is explained. In Section 5, Distributed Particle Filtering is proposed for fusing the state estimates produced by the distributed (local) processing units. In Section 6, fault diagnosis based on the Generalized Likelihood Ratio (GLR) is analyzed both for the case of distributed nonlinear Kalman Filtering and for the case of distributed Particle Filtering. In Section 7, simulation experiments are carried out to evaluate the performance of the distributed Kalman Filters and of the distributed Particle Filter in estimating the state of the monitored system (e.g. UAV sensors and actuators) over a wireless sensor network and subsequently in performing fault diagnosis. Section 9 concludes this chapter.
2 The problem of distributed fault diagnosis

2.1 Centralized vs distributed state estimation and fault diagnosis

This research work aims at developing and analyzing new solutions to the problem of distributed estimation for condition monitoring of nonlinear dynamical systems (e.g. automatic ground vehicles, unmanned surface or underwater vessels and unmanned aerial vehicles), so as to enable early detection of faults and the take up of efficient restoration measures. It is important to identify quickly the existence and position of a fault in autonomous vehicles and UAVs so as to carry out appropriate actions and to preserve the secure operation of such systems. To monitor the condition and to detect faults in these systems, some form of estimation is required, that will provide a model of the system’s operation in the fault-free condition.

Most of the existing state estimation methods used in fault diagnosis (e.g. for sensors and actuators in autonomous vehicles and UAVs) are centralized, which means that all sensing data are collected and processed at one unit. This not only causes excessive communication and computational burdens, but also creates a single point of failure. If the operation of the central information processing unit is affected by disturbances then the complete condition monitoring system may go out of order. Finally, most state estimation methods consider linear system models. This is also an approximation which is valid at specific operating points, while in practice the dynamics of the aforementioned systems are nonlinear.

To overcome the aforementioned flaws, the development of distributed nonlinear state estimation and distributed fault detection and isolation (FDI) tools is proposed. The first stage for distributed nonlinear state estimation is the processing of the collected measurements by suitable nonlinear stochastic estimation algorithms and the fusion of local state estimates into an aggregate state vector. Next, the aggregate state vector can be used by fault detection and isolation algorithms.

2.2 Localized state estimation in nonlinear robotic systems

**Nonlinear Kalman Filtering:** Two main approaches for state estimation in nonlinear dynamical systems are the Extended Kalman Filter and the Unscented Kalman Filter. The Extended Kalman Filter is the linearization of the standard Kalman Filter state estimation algorithm to the nonlinear case. The standard Kalman Filter is an optimal state observer in the sense that it can compensate in optimal way for the effect that process and measurement noises have on the estimation of the system’s state vector. The Extended Kalman Filter is based on a local linearization of the nonlinear dynamical model around the current state estimation and uses Jacobian matrices in the place of the state transition matrix and measurement matrix of the standard Kalman Filter. This first-order local linearization procedure introduces approximation errors due to the truncation of higher order term that appear in the associated Taylor series expansion (Rigatos and Tzafestas, 2007).

The Unscented Kalman Filter overcomes the flaws of the Extended Kalman Filter. Unlike EKF no analytical Jacobians of the system equations need to be calculated. This makes the UKF approach suitable for application in black-box models where analytical expressions of the system dynamics are either not available or not in a form that allows easy linearization. This is achieved through a different approach for calculating the posterior 1st and 2nd order statistics of a random variable that undergoes a nonlinear transformation. The state distribution is represented again by a Gaussian Random Variable but is now specified using a minimal set of deterministically chosen weighted sample points (Julier et al., 2000; Julier, 2004).

**Particle Filtering:** Particle filtering has improved performance over the established nonlinear filtering approaches (e.g. the EKF), since it can provide optimal estimation in nonlinear non-Gaussian state-space models, as well as in the estimation of nonlinear models. Particle filters can estimate the system states sufficiently when the number of particles (estimations of the state vectors which evolve in parallel) is large. Particle filtering has been mainly applied to state estimation for nonlinear systems fault diagnosis. The particle filtering algorithm is
reminiscent of evolutionary algorithms where a number of particles is subject to a mutation mechanism which corresponds to the prediction stage, and to selection mechanism which corresponds to the correction stage (Rigatos, 2009b).

2.3 Distributed state estimation in nonlinear robotic systems

Distributed state estimation is a rather new topic in the area of modelling and condition monitoring of autonomous vehicles and UAVs. More recently some approaches to distributed state estimation have been developed, this time taking into account transient phenomena in the system dynamics. Considering a linear system dynamics the Information Filter has been implemented for distributed estimation of robotic systems. Moreover, the distributed particle filter has been proposed in (Mohammadi and Asif, 2009) as a suitable methodology for decentralized state estimation of robotic systems. On the other hand distributed state estimation methodologies for nonlinear dynamical systems, such as the Extended Information Filter and the Unscented Information Filter have been tested in some problem of decentralized state estimation for robotic systems, such as autonomous vehicles and UAVs. The current chapter, offers new results on the use of the aforementioned distributed state estimation methods, i.e. Extended Information Filter (EIF), Unscented Information Filter and Distributed Particle Filter, in the case of nonlinear dynamical systems.

**Distributed Kalman Filtering**: The Extended Information Filter (EIF) and the Unscented Information Filter (UIF) can be proposed as possible approaches for fusing the state estimates provided by the local monitoring stations, under the assumption of Gaussian noises (Rigatos, 2010; Rigatos, 2011). The Extended Information Filter is a generalization of the Information Filter in which the local filters do not exchange raw measurements but send to an aggregation filter their local information matrices (local inverse covariance matrices) and their associated local information state vectors (products of the local information matrices with the local state vectors). In the case of the Unscented Information Filter there is no linearization of the power system observation equation. However the application of the Information Filter algorithm is possible through an implicit linearization which is performed by approximating the Jacobian matrix of the system’s output equation with the product of the inverse of the state vector’s covariance matrix (information matrix) with the cross-correlation covariance matrix between the system’s state vector and the system’s output. Again, the local information matrices and the local information state vectors are transferred to an aggregation filter which produces the global estimation of the system’s state vector.

**Distributed Particle Filtering**: The Distributed Particle Filter (DPF) is also proposed for fusing the state estimates provided by the local monitoring stations (local filters). The reason for using DPF is that it is well-suited to accommodate non-Gaussian measurements (Rigatos, 2011). A difficulty in implementing distributed particle filtering is that particles from one particle set (which correspond to a local particle filter) do not have the same support (do not cover the same area and points on the samples space) as particles from another particle set (which are associated with another particle filter). This can be resolved by transforming the particles sets into Gaussian mixtures, and defining the global probability distribution on the common support set of the probability density functions associated with the local filters.

2.4 Distributed state estimation for GPS failure detection

The GPS is a complex system used that is widely used in transportation and autonomous navigation. Therefore, for economy and safety reasons, it is particularly important to develop efficient tools for monitoring the condition of the GPS and detecting failures that may appear in several stages of its operation (Oshieng et al., 2004). The following GPS failure modes can be distinguished:

1. System level: System level failures can be related to erroneous clock behavior, incorrect modelling and
malfunction of the master control center, satellite payload performance, space vehicle performance, and RF performance. These errors are summarized in the following:

2. Erroneous clock behavior: Clock failures are one of the most common GPS failures. Satellite specific clock misbehavior is often hard to detect and can result in excessive code and carrier noise up to range errors of several thousand meters.

3. Erroneous modelling of the satellite orbits: Incorrect modelling of orbital parameters can result in the broadcast of incorrect satellite coordinates. This implies wrong satellite altitudes and leads finally to wrong range measurements. Moreover there are failures related to satellite orbits: these can be caused by satellite trajectory changes and instabilities in the satellite’s altitude.

4. Performance failures due to satellite payload: Erroneous or corrupted navigation data can be due to ionisation of the silicon material in memory devices. This affects the execution of code at the satellite’s processors and can lead to degraded navigation performance. To monitor quality of navigation data stored in memory, satellites reset their processors periodically.

5. Performance failures related to the space vehicle system: Degraded attitude control systems, excessive solar interference and power fluctuations lead to range errors due to malfunctioning hardware devices. This in turn causes increased signal-to-noise ratio and causes incorrect range measurements.

6. RF related performance failures: Onboard RF filters failures lead to corrupted side lobes. Moreover, there can be onboard multi-path and on-board signal reflections, de-synchronisation between data modulation and code and onboard interferences as well as inter-channel bias. These can result to corruption of the transmitted spectrum and range errors up to several meters.

7. Intended interference: it can take the form of jamming and spoofing. Jamming is the emission of sufficiently powerful power frequency energy close to the GPS spectrum. This can prevent GPS receivers from tracking the signal and cause significant position errors. Spoofing is the intended injection of false GPS-like signal causing significant navigation errors.

8. Unintended RF interference: This can be interference from RF transmitters emitting unwanted signal power in the L1/L2 band (e.g. ultra wide-band radar and communications broadcast television, VHF, personal electronic devices and mobile satellite services). This might lead the receivers to have difficulty in tracking the GPS signal to lose lock.

9. Failures related to sudden changes in the signal propagation properties. Small-scale (spatial and temporal) electron density fluctuations, especially in periods of high solar activity may affect the GPS signals significantly. The ionospheric effect might result in range errors up to 100m. Moreover, the troposphere has the effect of bending and refracting (delaying) the navigation signal. This can cause position errors between 2 and 25m. Finally, there are multi-path errors that result from reflection of the navigation signal on surfaces. These can result in position deviations of several hundred of meters.

10. Receiver and user-related performance failures: Even GPS receivers designed for the civil aviation can be subject to faults. Receivers shutdown, pause suddenly or even provide seriously incorrect positions. These failures can be attributed to: (i) power system failures or power fluctuations, (ii) software incompatibilities, (iii) overheating, (iv) instabilities in the quartz frequency standards, (v) receiver interface outages, (vi) receiver outages related to excessive electromagnetic activities (e.g. lighting) (vii) hardware incompatibilities if the GPS is coupled with other means of navigation (i.e. compasses, external clocks, air data, navigation data bases).
The concept of the chapter is that to deduce the existence of a failure in the GPS there should be comparison of the position measurements provided by the GPS against a reference signal that will be generated with the use of a distributed state estimation scheme. For instance, in the case of autonomous vehicles and UAVs one can assume that the reference signal is provided by fusing the state estimates computed at distributed information processing units (local filters). The distribute state estimation takes place in two levels (i) at the higher level fusion of the local state estimates is performed with the use of distributed state estimation approaches, such as the Extended Information Filter, the Unscented Information Filter or the Distributed Particle Filter, (ii) at the lower level local state estimates are generated by local nonlinear filters, such as Extended Kalman Filters, Unscented Kalman Filters or Particle Filters. The latter, provide local state estimates through the fusion of the state vector of the monitored AGV or UAV (cartesian coordinates and orientation) with the distance of the AGV or UAV from a reference surface. The cartesian coordinates and the orientation of the AGV or UA V can be obtained from IMUs, radars and gyrocompasses while the distance from the reference surface can be provided with the use of a vision sensor (camera), a sonar or again a radar.

Such a distributed state estimation scheme should finally operate over a wireless communication network. The proposed distributed state estimation scheme should take into account bandwidth limitations, communication delays and packet drops. Therefore, the information that is transmitted between the local information processing nodes (local filters) should be coded in such a manner that it requires minimum bandwidth, while robustness against transmission delays and measurement packet losses should be also assured.

### 3 EIF-based distributed estimation

#### 3.1 Extended Kalman Filtering

The distributed Extended Kalman Filter, also know as Extended Information Filter, performs fusion of the state estimates which are provided by local Extended Kalman Filters (Rigatos, 2010; Nettleton et al., 2003; Lee, 2008). Thus, the functioning of the local Extended Kalman Filters should be analyzed first. The following nonlinear state-space model is considered:

\[
\begin{align*}
    x(k+1) &= \phi(x(k)) + L(k)u(k) + w(k) \\
    z(k) &= \gamma(x(k)) + v(k)
\end{align*}
\]

where \(x \in \mathbb{R}^{m \times 1}\) is the system’s state vector and \(z \in \mathbb{R}^{p \times 1}\) is the system’s output, while \(w(k)\) and \(v(k)\) are uncorrelated, Gaussian zero-mean noise processes with covariance matrices \(Q(k)\) and \(R(k)\) respectively. The operators \(\phi(x)\) and \(\gamma(x)\) are \(\phi(x) = [\phi_1(x), \phi_2(x), \ldots, \phi_m(x)]^T\), and \(\gamma(x) = [\gamma_1(x), \gamma_2(x), \ldots, \gamma_p(x)]^T\), respectively. Following a linearization procedure, \(\phi\) is expanded into Taylor series about \(\hat{x}\), i.e.

\[
\phi(x(k)) = \phi(\hat{x}(k)) + J_\phi(\hat{x}(k))[x(k) - \hat{x}(k)] + \cdots
\]

where \(J_\phi(x)\) is the Jacobian of \(\phi\) calculated at \(\hat{x}(k)\). Likewise, \(\gamma\) is expanded about \(\hat{x}^- (k)\), i.e.

\[
\gamma(x(k)) = \gamma(\hat{x}^-(k)) + J_\gamma[x(k) - \hat{x}^-(k)] + \cdots
\]

where \(\hat{x}^- (k)\) is the estimation of the state vector \(x(k)\) before measurement at the \(k\)-th instant to be received and \(\hat{x}(k)\) is the updated estimation of the state vector after measurement at the \(k\)-th instant has been received. \(J_\gamma(x)\) is the Jacobian of \(\gamma\) calculated at \(\hat{x}(k)\). The resulting expressions create first order approximations of \(\phi\) and \(\gamma\). Thus the linearized version of the system is obtained:

\[
\begin{align*}
    x(k+1) &= \phi(\hat{x}(k)) + J_\phi(\hat{x}(k))[x(k) - \hat{x}(k)] + w(k) \\
    z(k) &= \gamma(\hat{x}^-(k)) + J_\gamma(\hat{x}^-(k))[x(k) - \hat{x}^-(k)] + v(k)
\end{align*}
\]
Now, the EKF recursion is as follows (Rigatos, 2009a):

- **Measurement update.** Acquire $z(k)$ and compute:

  \[
  K(k) = P^-(k)J^T_Y(\hat{x}^-(k)) \cdot [J_Y(\hat{x}^-(k))P^-(k)J^T_Y(\hat{x}^-(k)) + R(k)]^{-1} \\
  \hat{x}(k) = \hat{x}^-(k) + K(k)[z(k) - \gamma(\hat{x}^-(k))]
  \]

  \[
  P(k) = P^-(k) - K(k)J_Y(\hat{x}^-(k))P^-(k)
  \] (5)

- **Time update.** Compute:

  \[
  P^-(k+1) = J_\phi(\hat{x}(k))P(k)J^T_\phi(\hat{x}(k)) + Q(k) \\
  \hat{x}^-(k+1) = \phi(\hat{x}(k)) + L(k)u(k)
  \] (6)

3.2 EIF for local state estimates fusion

Again the discrete-time nonlinear system of Eq. (1) is considered. The Extended Information Filter (EIF) performs fusion of the local state vector estimates which are provided by the local Extended Kalman Filters, using the Information matrix and the Information state vector (Lee, 2008; Vercauteren, 2005). The Information Matrix is the inverse of the state vector covariance matrix, and can be also associated to the Fisher Information matrix (Rigatos and Zhang, 2009). The Information state vector is the product between the Information matrix and the local state vector estimate

\[
Y(k) = P^{-1}(k) = I(k) \\
\dot{y}(k) = P^{-1}(k)\dot{x}(k) = Y(k)\dot{x}(k)
\] (7)

The update equation for the Information Matrix and the Information state vector are given by

\[
Y(k) = P^-(k)^{-1} + J^T_Y(k)R^{-1}(k)J_Y(k) = Y^-(k) + I(k) \\
\dot{y}(k) = \dot{y}^-(k) + J^T_Y(k)R^{-1}(k)[z(k) - \gamma(x(k))] + J_Y(\dot{\hat{x}}^-) = \dot{y}^-(k) + i(k),
\] (8)

where

\[
i(k) = J^T_Y(k)R^{-1}(k)[z(k) - \gamma(x(k))] + J_Y(\dot{\hat{x}}^-)
\]

is the information state contribution

The predicted information state vector and Information matrix are obtained from

\[
\dot{y}^-(k) = P^-(k)^{-1}\dot{x}^-(k), \quad Y^-(k) = P^-(k)^{-1} = [J_\phi(k)P^-(k)J^T_\phi(k)] + Q(k)]^{-1}.
\] (10)

The outputs of the local filters are treated as measurements which are fed into the aggregation fusion filter (Lee, 2008; Vercauteren, 2005). Then each local filter is expressed by its respective error covariance and estimate in terms of information contributions and is described by

\[
P_i^{-1}(k) = P_i^{-1}(k)^{-1} + J^T_i(k)R^{-1}(k)J_i(k) \\
\hat{x}_i(k) = P_i(k)(P_i^{-1}(k)^{-1}\hat{x}^-_i(k)) + J^T_i(k)R^{-1}(k)[z_i(k) - \gamma_i(x(k))] + J_i(k)\dot{\hat{x}}^-_i(k).
\] (11)

The global estimate and the associated error covariance for the aggregate fusion filter can be rewritten in terms of the computed estimates and covariances from the local filters using the relations

\[
J^T_Y(k)R(k)^{-1}J_Y(k) = P_i^{-1}(k) - P_i^{-1}(k)^{-1} \\
J^T_Y(k)R(k)^{-1}[z_i(k) - \gamma_i(x(k))] + J_i(k)\dot{\hat{x}}^-_i(k) = P_i^{-1}(k)^{-1}\dot{x}_i(k) - P_i^{-1}(k)^{-1}\dot{x}_i(k-1).
\] (12)
For the general case of $N$ local filters $i = 1, \cdots, N$, the distributed filtering architecture is described by the following equations

$$
P(k)^{-1} = P^-(k)^{-1} + \sum_{i=1}^{N} [P_i(k)^{-1} - P_i^-(k)^{-1}]$$

$$
\dot{x}(k) = P(k)[P^-(k)^{-1}\dot{x}^-(k) + \sum_{i=1}^{N} P_i(k)^{-1}\dot{x}_i(k) - P_i^-(k)^{-1}\dot{x}_i^-(k)]
$$

(13)

The global state update equation in the above distributed filter can be written in terms of the information state vector and of the information matrix, i.e.

$$
\hat{y}(k) = \hat{y}^-(k) + \sum_{i=1}^{N} (\hat{y}_i(k) - \hat{y}_i^-(k)) \quad \hat{Y}(k) = \hat{Y}^-(k) + \sum_{i=1}^{N} (\hat{Y}_i(k) - \hat{Y}_i^-(k))
$$

(14)

The local filters provide their own local estimates and repeat the cycle at step $k + 1$. In turn the global filter can predict its global estimate and repeat the cycle at the next time step $k + 1$ when the new state $\hat{x}(k + 1)$ and the new global covariance matrix $P(k + 1)$ are calculated. From Eq. (13) it can be seen that if a local filter (processing station) fails, then the local covariance matrices and the local state estimates provided by the rest of the filters will enable an accurate computation of the target’s state vector.

### 4 UIF-based distributed estimation

#### 4.1 Unscented Information Filtering

The nonlinear state model of Eq. (1) is considered again. The Unscented Information Filter (UIF) can also perform fusion of the state vector estimates which are provided by local Unscented Kalman Filters running on the mobile robots, by weighting these estimates with local Information matrices (inverse of the local state vector covariance matrices). First, an augmented state vector $x_{\alpha}^- (k)$ is considered, along with the process noise vector, and the associated covariance matrix is introduced: $\dot{x}_{\alpha}^- (k) = [\dot{x}^- (k), \dot{w}^- (k)]^T$ and $P_{\alpha}^- (k) = diag\{P^- (k), Q^- (k)\}$.

As in the case of local (lumped) Unscented Kalman Filters, a set of weighted sigma points $X_{\alpha}^- (k)$ is generated as

$$
X_{\alpha, 0}^- (k) = \hat{x}_{\alpha}^- (k) \\
X_{\alpha, i}^- (k) = \hat{x}_{\alpha}^- (k) + [\sqrt{(n_{\alpha} + \lambda)P_{\alpha}^- (k - 1)}]_i, \quad i = 1, \cdots, n \\
X_{\alpha, i}^- (k) = \hat{x}_{\alpha}^- (k) - [\sqrt{(n_{\alpha} + \lambda)P_{\alpha}^- (k - 1)}]_i, \quad i = n + 1, \cdots, 2n
$$

(15)

where $\lambda = \alpha^2(n_{\alpha} + \kappa) - n_{\alpha}$ is a scaling, while $0 \leq \alpha \leq 1$ and $\kappa$ are constant parameters. The corresponding weights for the mean and covariance are defined as in the case of the lumped Unscented Kalman Filter

$$
W_i^{(m)} = \frac{\lambda}{n_{\alpha} + \kappa} \\
W_i^{(c)} = \frac{\lambda}{2(n_{\alpha} + \kappa)} + (1 - \alpha^2 + \beta)
$$

(16)

where $i = 1, \cdots, 2n_{\alpha}$ and $\beta$ is again a constant parameter (Rigatos, 2010; Julier, 2004). The equations of the prediction stage (measurement update) of the information filter, i.e. the calculation of the information matrix and the information state vector are (Lee, 2008; Vercauteren, 2005):

$$
\hat{y}^- (k) = Y^- (k) \sum_{i=0}^{2n_{\alpha}} W_i^{(m)} X_i^w (k) \\
Y^- (k) = P^- (k)^{-1}
$$

(17)

where $X_i^w$ are the predicted state vectors when using the sigma point vectors $X_i^w$ in the state equation $X_i^w (k + 1) = \phi(X_i^w(k)) + L(k)U(k)$. The predicted state covariance matrix is computed as

$$
P^- (k) = \sum_{i=0}^{2n_{\alpha}} W_i^{(c)} [X_i^w (k) - \hat{x}^- (k)][X_i^w (k) - \hat{x}^- (k)]^T
$$

(18)
In the equations of the Unscented Kalman Filter (UKF) there is no linearization of the system dynamics, thus the UKF cannot be included directly into the Extended Information Filter (EIF) equations (Lee, 2008). Instead, it is assumed that the nonlinear measurement equation of the system given in Eq. (1) can be mapped into a linear function of its statistical mean and covariance, which makes possible to use the information update equations of the EIF. Denoting $Y_i(k) = \gamma(X^i_t(k))$ (i.e. the output of the system calculated through the propagation of the $i$-th sigma point $X^i$ through the system’s nonlinear equation) the observation covariance and its cross-covariance are approximated by

$$P_{YY}(k) = E[(z(k) - \hat{z}(k)^{-})(z_k - \hat{z}(k)^{-})^T] \approx J_\gamma(k)P^-(k)J_\gamma(k)^T$$

$$P_{XY}(k) = E[(x_k - \hat{x}(k)^{-}) (z_k - \hat{z}(k)^{-})^T] \approx P(k)J_\gamma(k)^T$$

(19)

where $z(k) = \gamma(x(k))$ and $J_\gamma(k)$ is the Jacobian of the output equation $\gamma(x(k))$. Next, multiplying the predicted covariance and its inverse term on the right side of the information matrix and replacing $P(k)J_\gamma(k)^T$ with $P_{XY}(k)$ gives the following representation of the information matrix (Lee, 2008; Vercauteren, 2005):

$$I(k) = J_\gamma(k)^T R(k)^{-1}J_\gamma(k) = P^-(k)^{-1}P^-(-k)J_\gamma(k)^T R(k)^{-1}J_\gamma(k) P^-(k)^T (P^-(k)^{-1})^T = P^-(k)^{-1}P_{XY}(k) R(k)^{-1} P_{XY}(k)^T (P^-(k)^{-1})^T$$

(20)

where the cross-correlation matrix $P_{XY}(k)$ is calculated from

$$P_{XY}(k) = \sum_{i=0}^{2n_x} W^{(c)}_i [X_i^t(k) - \hat{x}^-(k)][Y_i(k) - \hat{z}^-(k)]^T$$

(21)

where $Y_i(k) = \gamma(X_i^t(k))$ and the predicted measurement vector $\hat{z}^-(k)$ is obtained by $\hat{z}^-(k) = \sum_{i=0}^{2n_x} W^{(m)}_i Y_i(k)$. Similarly, the information state vector $i(k)$ can be rewritten as

$$i(k) = J_\gamma(k)^T R(k)^{-1}[z(k) - \gamma(x(k)) + J_\gamma(k)^T \hat{x}^-(k)]$$

(i.e.

$$i(k) = P^-(k)^{-1}P_{XY}(k) R(k)^{-1} [z(k) - \gamma(x(k)) + P_{XY}(k)(P^-(k)^{-1})^T \hat{x}^-(k)]$$

(22)

A ”measurement” matrix $H^T(k)$ is defined as

$$H(k)^T = P^-(k)^{-1}P_{XY}(k)$$

(23)

while the information contributions equations are written as

$$i(k) = H^T(k) R(k)^{-1} [z(k) - \gamma(x(k)) + H(k) \hat{x}^-(k)]$$

$$I(k) = H^T(k) R(k)^{-1} H(k)$$

(24)

The above procedure leads to an implicit linearization in which the nonlinear measurement equation of the system given in Eq. (1) is approximated by the statistical error variance and its mean $z(k) = \gamma(x(k)) \approx H(k) x(k) + \tilde{u}(k)$, where $\tilde{u}(k) = \gamma(\hat{x}^-(k)) - H(k) \hat{x}^-(k)$ is a measurement residual term.

### 4.2 UIF for local state estimates fusion

It is assumed that the local Unscented Kalman Filters do not have access to each other row measurements and are allowed to communicate only their information matrices and their local information state vectors. Then Eq. (17) gives

$$P_i(k)^{-1} = P^-_{i}(k)^{-1} + H_i^T(k) R_i(k)^{-1} H_i(k)$$

$$\hat{x}_i = P_i(k)(P^-_{i}(k) \hat{x}^-_{i}(k) + H_i^T(k) R_i(k)^{-1} [z_i(k) - \gamma(x(k)) + H_i(k) \hat{x}^-(k)])$$

(25)
Each local information state contribution $i_i$ and its associated information matrix $I_i$ at the $i$-th filter are rewritten in terms of the computed estimates and covariances of the local filters, i.e.

$$H_i^T(k)R^{-1}H_i(k) = P_i^{-1}(k) - P_i^{-1}(k)$$

$$H_i^T(k)R_i(k)^{-1}[z_i(k) - y_i(x(k))] + H_i(k)\hat{x}_i(k)] = P_i(k^{-1}\hat{x}_i(k) - (P_i^{-1}(k^{-1}\hat{x}_i(k))$$

where according to Eq.(23) it holds $H_i(k) = P_i^{-1}(k)^{-1}P_{X_iY_i}(k)$. Next, the aggregate estimates of the UIF, can be written in terms of covariances (Lee, 2008; Vercauteren, 2005):

$$P(k)^{-1} = P^{-1}(k) + \sum_{i=1}^{N}[P_i(k)^{-1} - P_i^{-1}(k)]$$

$$\hat{x}(k) = P(k)[P^{-1}(k)^{-1}\hat{x}^{-}(k) + \sum_{i=1}^{N}(P_i(k)^{-1}\hat{x}_i(k) - P_i^{-1}(k)^{-1}\hat{x}_i^{-}(k))]$$

and also in terms of the information state vector and of the information matrix, i.e.

$$\hat{y}(k) = \hat{y}^{-}(k) + \sum_{i=1}^{N} (\hat{y}_i(k) - \hat{y}_i^{-}(k))$$

$$Y(k) = Y^{-}(k) + \sum_{i=1}^{N} Y_i(k) - Y_i^{-}(k)]$$

\section{DPF-based distributed estimation}

\subsection{Particle Filtering at local processing units}

In Particle Filtering (PF) a samples set of size $N$ is assumed, i.e. $N$ i.i.d. (independent identically distributed) variables $\xi_1, \xi_2, \ldots, \xi_N$. This sampling follows the p.d.f. $p(x)$ i.e. $\xi_1,\ldots,\xi_N \sim p(x)$. Instead of $p(x)$ the function $p(x) \approx p'(x) = \frac{1}{N} \sum_{i=1}^{N} \delta_{\xi_i}(x)$ can be used. It is assumed that all points $\xi_i$ have an equal weighted contribution to the approximation of $p(x)$. The PF recursion is (Rigatos, 2010):

- **Measurement update**: Acquire $z(k)$ and compute the new value of the state vector

$$p(x(k)|z) = \sum_{i=1}^{N} w_k^{i} \delta_{\xi_k}(x(k))$$

with corrected weights $w_k^{i} = \frac{w_k^{i} p(z(k)|x(\xi_k^{i}))}{\sum_{i=1}^{N} w_k^{i} p(z(k)|x(\xi_k^{i}))}$ and $\xi_k = \xi_k^{i}$

- **Resampling**: for substitution of degenerated particles

- **Time update**: compute state vector $x(k+1)$ according to the pdf

$$p(x(k+1)|z) = \sum_{i=1}^{N} w_k^{i} \delta_{\xi_k}(x(k))$$

where $\xi_k \sim p(x(k+1)|x(\xi_k^{i}))$.

\subsection{DPF for state estimates fusion}

The Distributed Particle Filter performs fusion of the state vector estimates which are provided by the local Particle Filters. This is succeeded by fusing the discrete probability density functions of the local Particle Filters into a common probability distribution of the system’s state vector, and which is given by (Ong et al., 2008):

$$p(x(k)|z_A \bigcup z_B) \propto \frac{p(x(k)|z_A) p(x(k)|z_B)}{p(x(k)|z_A \bigcap z_B)}$$

where $z_A$ is the sequence of measurements associated with the $i$-th processing unit and $z_B$ is the sequence of measurements associated with the $j$-th measurement unit. In the implementation of distributed particle filtering, the following issues arise: (i) Particles from one particle set do not have the same support (do not cover the
same area and points on the samples space) as particles from another particle set. Therefore a point-to-point application of Eq. (31) is not possible, (ii) The communication of particles representation (i.e. local particle sets and associated weight sets) requires significantly more bandwidth compared to other representations, such as Gaussian mixtures.

Fusion of the estimates provided by the local particle filters (located at different processing units) can be performed through the following stages. First, the discrete particle set of Particle Filter $A$ (Particle Filter $B$) is transformed into a continuous distribution by placing a Gaussian kernel over each sample (Fig. 1) $K_h(x) = h^2K(x)$, where $K()$ is the rescaled Kernel density and $h > 0$ is the scaling parameter (Musso et al., 2001). Then the continuous distribution $A$ ($B$) is sampled with the other particles set $B$ ($A$) to obtain the new importance weights, so that the weighted sample corresponds to the numerator of Eq. (31) (Fig. 2). Such a conversion from a discrete particle probability distribution function

$$\sum_{i=1}^{N} w_A^{(i)} \delta(x_A^{(i)})$$

into continuous distributions is denoted as

$$\sum_{i=1}^{N} w_A^{(i)} \delta(x_A^{(i)}) \rightarrow p_A(x) \quad (\sum_{i=1}^{N} w_B^{(i)} \delta(x_B^{(i)})) \rightarrow p_B(x)$$

(Fig. 1: Conversion of the particles discrete probability density function to a continuous distribution, after allocating a Gaussian kernel over each particle

According to Eq. (31), the common information appearing in the processing units $A$ and $B$ should not be taken into account. This can be succeeded through Monte Carlo sampling and suitable selection of the so-called "proposal distribution" (Ong et al., 2008). Thus, one can draw $N$ i.i.d samples from the associated probability density function $p(x)$, such that the target density is approximated by a point-mass function of the form $p(x) \approx \sum_{i=1}^{N} w_k^{(i)} \delta(x_k^{(i)})$, where $\delta(x_k^{(i)})$ is a Dirac delta mass located at $x_k^{(i)}$. Then the expectation of some function $f(x)$ with respect to the pdf $p(x)$ is given by

$$I(f) = E_{p(x)}[f(x)] = \int f(x)p(x)dx$$

The Monte-Carlo approximation of the integral with samples is then

$$I_N(f) = \frac{1}{N} \sum_{i=1}^{N} f(x^{(i)}) \text{ where } x^{(i)} \sim p(X) \text{ and } I_N(f) \rightarrow I(f) \text{ for } N \rightarrow \infty.$$
Since, the true probability distribution $p(x)$ is hard to sample from, the concept of importance sampling is to select a proposal distribution $\bar{p}(x)$ in place of $p(x)$, assuming that $\bar{p}(x)$ includes the support space of $p(x)$. Then the expectation of function $f(x)$, is calculated as

$$I(f) = \int f(x) \frac{p(x)}{\bar{p}(x)} \bar{p}(x) dx = \int f(x) w(x) \bar{p}(x) dx$$  \hspace{1cm} (36)

where $w(x)$ are the importance weights $w(x) = \frac{p(x)}{\bar{p}(x)}$. Then the Monte-Carlo estimation of the mean value of function $f(x)$ becomes $I_N(f) = \sum_{i=1}^{N} f(x^{(i)}) w(x^{(i)})$. For the division operation, the desired probability distribution is

$$p(x^{(i)}) = \frac{p_A(x^{(i)})}{p_B(x^{(i)})}$$  \hspace{1cm} (37)

In that case the importance weights of the fused p.d.f. become

$$w(x^{(i)}) = \frac{p_A(x^{(i)})}{p_B(x^{(i)})} \bar{p}(x^{(i)})$$  \hspace{1cm} (38)

which is then normalized so that $\sum_{i=1}^{N} w(x^{(i)}) = 1/N$, where $N$ is the number of particles. The next step is to decide what will be the proposal distribution $\bar{p}(x)$. An option is to take $\bar{p}(x)$ to be a uniform distribution, with a support that covers both of the support sets of the distributions A and B, i.e. $\bar{p}(x) = U(x)$. Then the sample weights $\bar{p}(x^{(i)})$ are all equal at a constant of value $C$. Hence the importance weights are

$$w(x^{(i)}) = \frac{p_A(x^{(i)})}{\bar{p}(x^{(i)})C}.$$  \hspace{1cm} (39)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Fusion of the p.d.f. produced by the local particle filters}
\end{figure}
6 Fault detection based on likelihood ratio

6.1 Change detection through the likelihood ratio

Fault diagnosis consists of two stages: (i) residual generation, i.e. a signal that shows the difference between the measured and the estimated value of the monitored parameter, (ii) fault threshold selection so as to detect incipient faults while reducing the false alarms rate (Ding, 2008; Isermann, 2006). The most generic fault threshold is derived from the likelihood ratio (LR) and this has been applied to several FDI problems (Li and Kadirkamanathan, 2001; Basseville and Nikiforov, 1993). If the observations \( y_k, k = 1, 2, \ldots, N \) are i.i.d, following the probability density \( p(\theta)(y) \), then joint likelihood ratio for the observations from \( y_j \) to \( y_k \) can be expressed as

\[
S_{k|j} = \ln \frac{p(y_j, \ldots, y_k|\theta_1)}{p(y_j, \ldots, y_k|\theta_0)}
\]

which due to statistical independence equivalently can be written as

\[
S_{k|j} = \sum_{i=j}^{k} s_i \quad \text{and} \quad s_i = \ln \frac{p_{\theta_1}(y_i)}{p_{\theta_0}(y_i)}
\]

Thus taking into account the statistical independence it holds

\[
p(y_j, \ldots, y_k|\theta_1) = p(y_j|\theta_1)p(y_{j+1}|\theta_1)\cdots p(y_k|\theta_1).
\]

Similarly, one has

\[
p(y_j, \ldots, y_k|\theta_0) = p(y_j|\theta_0)p(y_{j+1}|\theta_0)\cdots p(y_k|\theta_0).
\]

Assuming that \( \theta = \theta_0 \) before change, and \( \theta = \theta_1 \) after change, then the typical behavior of the joint cumulative likelihood ratio (LR) \( S_{k} \) shows on average a negative drift before change, and a positive drift after change.

6.2 Fault detection through the Generalized Likelihood Ratio

The detection problem, given the observations up to time \( k \), consists of testing between two hypotheses which can be written as:

No change hypothesis \( H_0 : j > k \) and
Change hypothesis \( H_1 : j \leq k \),

where \( j \) is the unknown change time. The likelihood ratio between these hypotheses is defined by Eq. (41). Replacing the unknown change time \( j \) by its maximum likelihood estimate (MLE) under \( H_1 \), the following change detector can be obtained:

\[
g_k = S_{k|j}^k = \max_{j \geq H_1} S_{j|k}^k \leq \lambda
\]

where \( g_k \) is the decision function and \( \lambda > 0 \) is a threshold. In other words, \( H_1 \) is decided whenever \( g_k \) exceeds \( \lambda \), and \( H_0 \) otherwise. If the parameter \( \theta_1 \) after change is unknown, then the cumulative LR defined in Eq. (41) is a function of two unknown independent parameters, namely the unknown change time \( j \) and the value of the parameter \( \theta_1 \) after change. In this case the cumulative likelihood ratio should be written as

\[
S_{j|k}^k(\theta_1) = \sum_{i=j}^{k} \ln \frac{p_{\theta_1}(y_i)}{p_{\theta_0}(y_i)}
\]
One of the solutions to the above change detection problem is to replace $\theta_1$ by its MLE, which results in the generalized likelihood ratio (GLR) algorithm. Thus, the decision function of the GLR change detector, which involves the double maximization is given by

$$g_k = \max_{1 \leq j \leq k} \sup_{\theta_1} S_j^k(\theta_1)$$  \hspace{1cm} (47)$$

For the purpose of FDI, and based on the Generalized Likelihood Ratio, optimal fault threshold selection has been analyzed in (Basseville and Nikiforov, 1993; Fouladirad and Nikiforov, 2005).

7 Particle Filtering for fault detection

It is assumed that the measurement noise $v$ has the same dimensionality as the measurement $y$ and the state $x_j$ and the measurement $y_j$ the measurement noise $y_j$ is defined by an observation-error function $v_j = g_j(y_j,x_j)$, where $g_j(.,.)$ has a Jacobian defined by

$$\frac{\partial g_j}{\partial y_j}$$  \hspace{1cm} (48)$$

The likelihood ratio for the hypothesized model is

$$S_k^j = \sum_{j=1}^k J_n \frac{p(y_j|H_1,Y_{j-1})}{p(y_j|H_0,Y_{j-1})}$$  \hspace{1cm} (49)$$

where the likelihood of the observation $y_j$ given its past values $Y_{j-1}$, i.e. $p(y_j|H_1,Y_{j-1})$, is the output estimation based on hypothesis $H_1$, and which is defined by the measurement model and the known statistics of the observation error function $e_j = g_j(y_j,x_j)$. If the pdf of $e_j$ is denoted by $q_j(e_j)$, then the probability $p(y_j|H_1,Y_{j-1})$ can be expressed in a way analogous to the LR residual (Li and Kadirkamanathan, 2001)

$$s_j = p(y_j|H_1,Y_{j-1}) = q_j(e_j)\left|\frac{\partial e_j}{\partial y_j}\right|$$  \hspace{1cm} (50)$$

where $x_{j|j-1}$ is the output estimation of the model, given $Y_{k-1}$. It can be verified that in the Gaussian case, the quantity defined by Eq. \(50\) is just the innovation likelihood, which can be derived from the Kalman filter equations. For the general nonlinear non-Gaussian model there is no analytical means to perform the calculation. However, with the particle filter this quantity can be estimated using the complete pdf information of the predicted state $x_{j|j-1}$ represented by the swarm of particles. Since $x_{j|j-1} : j = 1, \ldots, N$ can be considered as the samples from $p(x_j|H_1,Y_{j-1})$ for the model, the required quantity can be computed via the Monte-Carlo integration as follows:

$$s_j = p(y_j|H_1,Y_{j-1}) \approx \frac{1}{N} \sum_{i=1}^N p(y_j|x_{j|i-1})$$  \hspace{1cm} (51)$$

This means that taking $N$ particles, the likelihood ratio is given at time instant $i$ by

$$s_j(i) = \frac{1}{N} \sum_{j=1}^N w_j(i)$$  \hspace{1cm} (52)$$

where $w_j(i)$ are the unnormalized particle weights, of the particle estimator.
8 Simulation tests

8.1 Nonlinear control for UAV navigation

It is assumed that helicopter-like UAVs, perform manoeuvres at a constant altitude. One can obtain the following description for the UAV kinematics (Léchevin and Rabbath, 2006):

\[
\begin{align*}
\dot{x} &= v \cos(\theta), \\
\dot{y} &= v \sin(\theta), \\
\dot{\theta} &= \frac{v}{l} \tan(\phi)
\end{align*}
\]  
(53)

where using the analogous of the unicycle robot \( v \) is the velocity of the UAV, \( l \) is the UAV’s length, \( \theta \) is the UAV’s orientation (angle between the transversal axis of the UAV and axis \( OX \)), and \( \phi \) is a steering angle. Using flatness-based control theory one can define the flat output as the cartesian position of the UAV’s center of symmetry, denoted as \( \eta = (x, y) \) (Villagra et al., 2007). Then, the resulting dynamic compensator is (return to the initial control inputs \( v \) and \( \omega \))

\[
\begin{align*}
\dot{\xi} &= u_1 \cos(\theta) + u_2 \sin(\theta), \\
v &= \frac{\xi}{u_2 \cos(\theta) - u_1 \sin(\theta)}
\end{align*}
\]  
(54)

where

\[
\begin{align*}
u_1 &= \ddot{x}_d + k_{p_1} (x_d - x) + k_{d_1} (\dot{x}_d - \dot{x}) \\
u_2 &= \ddot{y}_d + k_{p_1} (y_d - y) + k_{d_1} (\dot{y}_d - \dot{y})
\end{align*}
\]  
(55)

and which results in the following error dynamics for the closed-loop system

\[
\begin{align*}
\dot{e}_x + k_d \dot{e}_x + k_{p_1} e_x &= 0 \\
\dot{e}_y + k_d \dot{e}_y + k_{p_2} e_y &= 0
\end{align*}
\]  
(56)

where \( e_x = x - x_d \) and \( e_y = y - y_d \). The proportional-derivative (PD) gains are chosen as \( k_{p_1} > 0 \) and \( k_{d_1} > 0 \) for \( i = 1, 2 \).

8.2 GPS integrity testing

The concept of the chapter is that to deduce the existence of a failure in the GPS there should be comparison of the position measurements provided by the GPS against a reference signal that will be generated with the use of a distributed state estimation scheme. For instance, in the case of autonomous vehicles and UAVs one can assume that the reference signal is provided by fusing the state estimates computed at distributed information processing units (local filters). The distributed state estimation takes place in two levels (i) at the higher level fusion of the local state estimates is performed with the use of distributed state estimation approaches, such as the Extended Information Filter, the Unscented Information Filter or the Distributed Particle Filter, (ii) at the lower level local state estimates are generated by local nonlinear filters, such as Extended Kalman Filters, Unscented Kalman Filters or Particle Filters. The latter, provide local state estimates through the fusion of the state vector of the monitored AGV or UAV (cartesian coordinates and orientation) with the distance of the AGV or UAV from a reference surface. The cartesian coordinates and the orientation of the AGV or UAV can be obtained from IMUs, radars and gyrocompases while the distance from the reference surface can be provided with the use of a vision sensor (camera), a sonar or again a radar.

It is considered that the previously analyzed helicopter model is monitored by \( n = 2 \) different ground stations. The distributed filtering architecture is shown in Fig. 3.

To perform integrity testing of the GPS it is necessary to compare the GPS measurements against the estimated coordinates of the UAV which are provided by distributed filtering and fusion of the the IMU measurements of the UAV with measurements from visual sensors (visual odometry) (Vissiere et al, 2008). The inertial coordinates system \( OXY \) is defined. Furthermore the coordinates system \( O'X'Y' \) is considered (Fig. 4). \( O'X'Y' \) results from \( OXY \) if it is rotated by an angle \( \theta \). The coordinates of the center of symmetry of
Figure 3: Integrity monitoring for the GPS using a distributed filtering scheme.

Figure 4: Reference frames for the UAV.
the UAV with respect to $OXY$ are $(x, y)$, while the coordinates of the GPS or visual sensor that is mounted on the UAV, with respect to $O'X'Y'$ are $x_i, y_i$. The orientation of the visual sensor with respect to $OXY$ is $\theta_i$. Thus the coordinates of the visual sensor with respect to $OXY$ are $(x_i, y_i)$ and its orientation is $\theta_i$, and are given by:

\[
x_i(k) = x(k) + x_i' \sin(\theta(k)) + y_i' \cos(\theta(k)) \\
y_i(k) = y(k) - x_i' \cos(\theta(k)) + y_i' \sin(\theta(k)) \\
\theta_i(k) = \theta(k) + \theta_i
\] (57)

The visual sensor $i$ is at position $x_i(k), y_i(k)$ with respect to the inertial coordinates system $OXY$ and its orientation is $\theta_i(k)$. Using the above notation, the distance of the visual sensor $i$, from the plane $P^j$ is represented by $P^j_i$, $P^j_{ni}$ where (i) $P^j_i$ is the normal distance of the plane from the origin $O$, (ii) $P^j_{ni}$ is the angle between the normal line to the plane and the x-direction: $d^j_i(k) = P^j_i - x_i(k) \cos(P^j_{ni}) - y_i(k) \sin(P^j_{ni})$ (see Fig. 4).

Results on the UIF and DPF performance in estimating the state vectors of multiple UAVs when observed by distributed processing units is given in Fig. 5 and Fig. 11, respectively.

The advantages of using Distributed Filtering are as follows: (i) there is robust state estimation (which in the DPF case is not constrained by the assumption of Gaussian noises). The fusion between the local filters compensates for deviations in state estimates (which in the UIF case can be due to linearization errors while in the DPF case can be due to outlier particles) thus resulting in an aggregate state distribution that confines with accuracy the real state vector of each UAV. (ii) If a local processing unit (local filter) fails, the reliability of the aggregate state estimation is preserved (iii) computation load can be better managed comparing to a centralized filtering architecture. The greatest part of the necessary computations is performed at the local filters. Moreover the advantage of communicating state posteriors over raw observations is bandwidth efficient, which is particularly useful for control over wireless sensor networks.

In case of a GPS fault one can observe deviation of the estimated UAV coordinates which are provided by the distributed filtering scheme from the UAV coordinates provided by the GPS.

**Figure 5:** UIF-based trajectory tracking (dashed line) of (a) a circular reference trajectory (b) a curve-shaped reference trajectory.
Figure 6: Deviation of the estimated UAV coordinates which are provided by the Extended Information Filter from the UAV coordinates provided by the GPS in case of a circular reference trajectory (a) coordinate $x$ (b) coordinate $y$.

Figure 7: Deviation of the estimated UAV coordinates which are provided by the Extended Information Filter from the UAV coordinates provided by the GPS in case of a curved-shaped reference trajectory (a) coordinate $x$ (b) coordinate $y$. 
Figure 8: UIF-based trajectory tracking (dashed line) of (a) a circular reference trajectory (b) a curve-shaped reference trajectory.

Figure 9: Deviation of the estimated UAV coordinates which are provided by the Unscented Information Filter from the UAV coordinates provided by the GPS in case of a circular reference trajectory (a) coordinate $x$ (b) coordinate $y$. 
Figure 10: Deviation of the estimated UAV coordinates which are provided by the Unscented Information Filter from the UAV coordinates provided by the GPS in case of a curved-shaped reference trajectory (a) coordinate x (b) coordinate y.

Figure 11: DPF-based trajectory tracking (dashed line) of (a) a circular reference trajectory (b) a curve-shaped reference trajectory.
Figure 12: Deviation of the estimated UAV coordinates which are provided by the Distributed Particle Filter from the UAV coordinates provided by the GPS in case of a circular reference trajectory (a) coordinate $x$ (b) coordinate $y$.

Figure 13: Deviation of the estimated UAV coordinates which are provided by the Distributed Particle Filter from the UAV coordinates provided by the GPS in case of a curved-shaped reference trajectory (a) coordinate $x$ (b) coordinate $y$. 
9 Conclusions

The chapter has examined the problem of distributed state estimation for condition monitoring of nonlinear dynamical systems. The concept of the chapter is that to decide about the existence of a failure in the GPS there should be comparison of the position measurements provided by the GPS against a reference signal that will be generated with the use of a distributed state estimation scheme. The chapter proposed the Extended Information Filter (EIF) the Unscented Information Filter (UIF) and the Distributed Particle Filter (DPF) as possible approaches for fusing the state estimates obtained by the local monitoring stations.

For instance, in the case of autonomous vehicles and UAVs one can assume that the reference signal is provided by fusing the state estimates computed at distributed information processing units (local filters). The distributed state estimation takes place in two levels (i) at the higher level fusion of the local state estimates is performed with the use of distributed state estimation approaches, such as the Extended Information Filter, the Unscented Information Filter or the Distributed Particle Filter, (ii) at the lower level local state estimates are generated by local nonlinear filters, such as Extended Kalman Filters, Unscented Kalman Filters or Particle Filters. The latter, provide local state estimates through the fusion of the state vector of the monitored AGV or UAV (cartesian coordinates and orientation) with the distance of the AGV or UAV from a reference surface. The cartesian coordinates and the orientation of the AGV or UAV can be obtained from IMUs, radars and gyrocompasses while the distance from the reference surface can be provided with the use of a vision sensor (camera), a sonar or again a radar.

The chapter proposed the Extended Information Filter (EIF) the Unscented Information Filter (UIF) and the Distributed Particle Filter (DPF) as possible approaches for fusing the state estimates obtained by the local monitoring stations, under the assumption of Gaussian noises. The Extended Information Filter is a generalization of the Information Filter in which the local filters do not exchange raw measurements but send to an aggregation filter their local information matrices (inverse covariance matrices, which can be also associated to the Fisher Information Matrices) and their associated local information state vectors (products of the local Information matrices with the local state vectors). For nonlinear system dynamics, such as the AGVs and UAVs, the calculation of the information matrices and information state vectors requires the linearization of the local observation equations in the system’s state space description and consequently the computation of Jacobian matrices is needed.

In the case of the Unscented Information Filter there is no linearization of the model of the monitored nonlinear dynamical system. However the application of the Information Filter algorithm is possible through an implicit linearization which is performed by approximating the Jacobian matrix of the system’s output equation by the product of the inverse of the state vector’s covariance matrix with the cross-covariance matrix between the system’s state vector and the system’s output. Again, the local information matrices and the local information state vectors are transferred to an aggregation filter which produces the global estimation of the system’s state vector.

As far as the fault diagnosis part is concerned the FDI method can be based on the combination of the Kalman or the particle filtering algorithm with the likelihood ratio test. The Kalman Filter-based fault diagnosis assumes Gaussian measurement statistics while the Particle Filter-based fault diagnosis is not subject to assumptions about the measurement error distribution. The residuals between the state vectors generated by Particle Filter and the Kalman Filter on the one side and the state vectors of the monitored system on the other side were calculated. The proposed FDI method, which is based on Particle Filtering, is simple and applicable to a quite general class of nonlinear systems. However, as in all particle filter applications intensive computations are required.

Comparing to centralized state estimation and fault diagnosis the proposed distributed state estimation schemes have significant advantages: (i) they are fault tolerant: if a local processing unit is subject to a fault then
state estimation is still possible and accurate, (ii) the aggregation performed on the local EKF, UKF or PF also compensates for deviations in state estimates of local filters (which in the EKF case can be due to linearization errors), (iii) communication overhead remains low even in the case of a large number of distributed measurement units, because the greatest part of state estimation is performed locally and only information matrices and state vectors are communicated between the local processing units.

References


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