A Mathematical Model on Glucose-Insulin Regulatory System with the Impact of Physical Activities

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Abstract

This work proposes a generalized Mathematical model used to study the effect of physical activities on glucose-insulin regulatory system for both non-diabetic person as well as the diabetic patient. We have considered glucose and generalized insulin variable in this mathematical model. The numerical results are obtained to show the behavior of glucose insulin regulatory system for non-diabetic as well as diabetic person with the impact of physical activities. The conclusion of this work suggests that the glucose level decreases by the effect of physical activities in daily life routine of a diabetic person.

Keyword: Physical activities, Glucose-insulin regulatory system, Mathematical model, diabetes mellitus, differential equations.

1 Introduction

The human body employs several mechanisms to regulate the concentration of glucose in the bloodstream. The rates of glucose uptake and release from specific organs within the body are modulated directly by the concentrations of metabolites and hormones, and indirectly by the autonomic nervous system. Physical exercise is an important factor in diabetes management. Regular physical activity, which can include structured exercise in a variety of forms, offers a net benefit for most individuals with diabetes.

Regular physical activity enhances insulin sensitivity, increases cardiorespiratory fitness, improves glycemic control, reduces the risk of cardiovascular mortality, and enhances psychosocial well-being. During exercise, the caloric needs of muscles are initially met by glucogenolysis in muscle and increased uptake of glucose. Plasma glucose initially rises with increase hepatic glucogenolysis but may fall with strenuous, prolon exercise. After exercise liver glycogen is replenished by additional glyconeogenesis and a decrease in hepatic glucose output. Exercise has direct effects in carbohydrate metabolism the entry of glucose into skeletal muscle is increased during insulin sensitivity of the muscle increase in sensitivity persists for several hours after exercise and regular exercise training can produce prolong increase in insulin sensitivity.

While evaluating the type, duration and intensity of physical exercises it is necessary to consider the level of physical state of patients. Diabetics perform aerobical and anaerobical exercises. Anaerobical exercises last less than 2 minutes, for example, short distance running, swimming or weight lifting. Aerobiocal exercises last over 2 minutes, for example, long distance running and other sporting fields [4].

During physical activities, whole-body oxygen consumption may increase by as much as 20-fold, and even greater raises may occur in the working muscles. To meet its energy needs under these circumstances, skeletal muscle exercise, at a greatly raised rate, its own stores of glycogen and triglycerides, as well as free fatty acids (FFAs) derived from the breakdown of adipose tissue triglycerides and glucose released from the liver [15].

Exercise can precipitate hypoglycemic in diabetes not only because of the increase in muscle uptake of glucose but also because absorption of injected insulin is more rapid during exercise. When the intensity of exercise is extremely high the plasma glucose level falls the fist symptom are palpitations, sweating and nervousness due to autonomic discharge at lower plasma glucose level [14]. People with type 1 are at the highest risk for hypoglycemia. People with type 2 are less likely to have issues with hypoglycemia during or after exercise, unless they are on insulin or an insulin secretagogue. In short, there are few ways that exercise lowers blood glucose:

• Insulin sensitivity is increased, so your cells are better able to use any available insulin to take up glucose during and after activity.

• When your muscles contract during activity, it stimulates another mechanism that is completely separate of insulin. This mechanism allows your cells to take up glucose and use it for energy whether insulin is available or not.
Many scientists have studied the nature and diagnosis of this disease; these studies are either experimental or theoretical [14]. The language of mathematics is widely used to describe a myriad of naturally occurring phenomena. This is partly due to the fact that mathematical modeling along with stability analysis allows for a qualitative description/understanding of systems in which one or more of the components are not sufficiently or accurately determined, as it is particularly the case in the medical sciences, where it is next to impossible to obtain data for many parameters. It can also help estimate parameters based on which dynamical behavior is expected from a given system, or it can aide in the understanding of which parameters are the most significant for the behavior.

The main contribution of our study is to explain the behavior of glucose-insulin regulatory system with impact of physical activities on a diabetic person from breakfast (8.00am) to lunch (1.00pm). This work is considered for non-insulin dependent patient.

### 2 Mathematical Model

Consider a mathematical model comprised of glucose level G and insulin level I. Many parameters have been taken which include the parameter of physical activities as well and on the basis of these parameters values a mathematical model is formed. The model is defined as:

\[
\frac{dG}{dt} = -(A_1 - a_1)G + A_2 I + G_s, \quad (1)
\]

\[
\frac{dI}{dt} = -(A_3 + A_4)I + A_5 G + I_s, \quad (2)
\]

All the variables and parameters values used in mathematical models are described as:

- \(G(t)\): The plasma glucose concentration at time \(t\) (mg/dl)
- \(I(t)\): The plasma insulin concentration at time \(t\) (μU/ml)
- \(G_s\): This is the basal value of plasma glucose (mg/dl)
- \(I_s\): This is the basal value of plasma insulin (μU/ml)
- \(A_1\): Insulin independent rate constant of glucose rate uptake in muscles, liver and adipose tissue (min-1).
- \(A_2\): The rate of decrease in tissue glucose uptake ability (min-1).
- \(A_3\): The insulin independent increase in glucose uptake ability in tissue per unit of insulin concentration \(1\) (min-2 (μU/ml)).
- \(A_4\): The first order decay rate for insulin in plasma (min-1) pancreatic \(\beta\)-cells release insulin
- \(A_5\): The rate of the pancreatic \(\beta\)-cells’ release of insulin after the glucose injection and with glucose concentration above \(h\) (μU/ml) min-2 (mg/dl-1)
- \(a_1\): Level of conversion of glycogen into glucose during physical activities (min-1).

### 3 Numerical Result

In equation (1) and (2) eliminating \(G\) and \(I\) step by step we get:

\[
D^2 G + w^2 DG + 2\alpha G = s',
\]

\[
D^2 I + w^2 DI + 2\alpha I = s',
\]

where

\[
w^2 = (A_1 - a_1)(A_3 + A_4) - A_2 A_3,
2\alpha = (A_1 - a_1)(A_3 + A_4),
S_1 = A_2 I_s + (A_3 + A_4)G_s,
S_2 = (A_1 - a_1) + A_3 G_s.
\]

If \(S_1(t) = e^{aw}\sin wt\) and \(S_2(t) = 0\), then

\[
G = e^{aw}\left(X \cos wt + Y \sin wt + \frac{B \sin wt}{w}\right),
\]

\[
I = e^{aw}\left(M \cos wt + N \sin wt\right).
\]

#### 3.1 Numerical solution for non-diabetic person

**Case 1 (a): If a non-diabetic person in doing physical activity in daily life routine**

By using these parameter values [7] and [13] we get \(\alpha = 0.014\), \(a_1 = 0\) and \(w^2 = 0.0019\) and hence the solution of equation (5) and (6) is given by:

\[
G = e^{0.014t}\left(X \cos 0.0435t + Y \sin 0.0435t + \frac{B \sin 0.0435t}{0.0435}\right),
\]

\[
I = e^{0.014t}\left(M \cos 0.0435t + N \sin 0.0435t\right).
\]

In figure 1(a) represents the behavior of glucose and insulin regulatory system of a non-diabetic person when a person is doing physical activities. It shows that glucose level of a non-diabetic person is 70 mg/dl at the time of breakfast and grows 135 mg/dl after 2.5 hours of breakfast then comes down to 110 mg/dl.

![Figure 1(a): A plot of glucose concentration (G) & insulin concentration (I) versus time for a non-diabetic person is not doing physical activities (\(a_1=0\)).](image-url)
Case 1 (b): If a non-diabetic person in some doing physical activity in daily life routine

By using these parameter values [7] and [13] we get $\alpha = 0.0048$, $a_1 = 0.010$ and $w^2 = 0.00038$ and hence the solution of equation (5) and (6) is given by:

\[
G = e^{0.0128t} \left( X \cos 0.0173t + Y \sin 0.0173t + \frac{B \sin 0.0173t}{0.0173} \right),
\]

\[
I = e^{0.0128t} \left( M \cos 0.0173t + N \sin 0.0173t \right).
\]

In figure 1(b) represents the behavior of glucose and insulin regulatory system of a non-diabetic person when a person is doing physical activities. It shows that glucose level of a non-diabetic person is 70 mg/dl at the time of breakfast and grows 129 mg/dl after 2.5 hrs. of breakfast then comes down to 95 mg/dl.

3.2 Numerical solution for diabetic patient

Case 2 (a): If a diabetic person in not doing physical activity in daily life routine

By using these parameter values [7] and [13] we get $\alpha = 0.0048$, $a_1 = 0.010$ and $w^2 = 0.00038$ and hence the solution of equation (5) and (6) is given by:

\[
G = e^{0.0128t} \left( X \cos 0.0173t + Y \sin 0.0173t + \frac{B \sin 0.0173t}{0.0173} \right),
\]

\[
I = e^{0.0128t} \left( M \cos 0.0173t + N \sin 0.0173t \right).
\]

In figure 2 (a) represents the behavior of glucose and insulin regulatory system of a diabetic patient when a patient is not doing physical activities. It shows that glucose level of a diabetic person is 150 mg/dl at the time of breakfast and grows continuously 245 mg/dl within 6 hrs. of breakfast.

Case 2 (b): If a diabetic person in some doing physical activity in daily life routine

By using these parameter values [7] and [13] we get $\alpha = 0.0038$, $a_1 = 0.0010$ and $w^2 = 0.000299$ hence the solution of equation (5) and (6) is given by:

\[
G = e^{0.0128t} \left( X \cos 0.0171t + Y \sin 0.0171t + \frac{B \sin 0.0171t}{0.0171} \right),
\]

\[
I = e^{0.0128t} \left( M \cos 0.0171t + N \sin 0.0171t \right).
\]

In figure 2 (b) represents the behavior of glucose and insulin regulatory system of a diabetic patient when a patient is doing physical activities. It shows that glucose level of a diabetic person is 150 mg/dl at the time of breakfast and grows continuously 245 mg/dl within 6 hrs. of breakfast.

4 Conclusion

The benefits of physical activity in reducing risks for various conditions—including cardiovascular disease, osteoporosis, and diabetes—are well documented [15]. Physical activity training is a well-established prevention strategy, treatment and management therapy for diabetic patients. Through this work we created a mathematical model, which shows the effect of very light physical activities (aerobic exercises...
which include-medium to long distance running/ jogging, swimming , cycling, and walking ) on the glucose-insulin regulatory system for both non-diabetic as well as diabetic person. It is clearly visible though graph that glucose concentration level comes down in each case and hence the persons have lesser risk to become diabetic.

Numerical solutions of the above figures represents the solutions of mathematical model for 5 hours i.e.(8.00am-1.00pm), which shows the physical activities prescribed by doctor will helpful for the cure of a diabetic patient.

For diabetic patients, the emphasis must be on adjusting the treatments regimen to allow safe participation in all forms of physical activities consistent with a patient’s requirements and goals. At last, all patients with diabetes should have the opportunity to benefit from the many valuable effects of physical activity.

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